KMA254 Past Exams. Question 1 (2010 Backwards)

2010

1. Find the general solutions to the following first-order ODEs:

(a) $\dfrac{dy}{dx} = \sqrt{1+y^2}$

   [HINT: It may be helpful to use $\displaystyle\int \dfrac{dy}{\sqrt{1+y^2}} = \operatorname{arcsinh}(y)$.]

   [6 points]

(b) $\dfrac{dy}{dx} + 3y = e^{-5x}$

   [6 points]

(c) $\dfrac{dy}{dx} = \left(\dfrac{y}{x}\right) - \dfrac{1}{3}\left(\dfrac{y}{x}\right)^4$

   [8 points]

2009

1. Find the general solutions to the following first-order ODEs:

(a) $\dfrac{dy}{dx} + 2y = e^{-2x}$

   [6 points]

(b) $\dfrac{dy}{dx} = 1 - y^2$

   [HINT: It may be helpful to use $\dfrac{1}{1-y^2} = \dfrac{1}{2}\left(\dfrac{1}{1-y} + \dfrac{1}{1+y}\right)$.]

   [7 points]

(c) $\dfrac{dy}{dx} + y = xy^2$

   [HINT: It may help to use $\displaystyle\int xe^{-x}dx = -(x+1)e^{-x}$.]

   [7 points]

2008

1. Find the general solutions to the following first-order ODEs:

(a) $\dfrac{dy}{dx} = \dfrac{y}{x^2}$

[6 points]

(b) $\dfrac{dy}{dx} + 2xy = x^3$

[HINT: It may help to use $\displaystyle\int x^3 e^{x^2}\, dx = \frac{1}{2}\left(x^2 - 1\right)e^{x^2}$.]

[7 points]

(c) $\dfrac{dy}{dx} = \dfrac{(y+x)^2}{(y+x)^2 + 1}$

[HINT: Try making the change of variable $u = y + x$.
It is not necessary to solve explicitly for y in terms of x .]

[7 points]

2007
1. Find the general solutions to the following first-order ODEs:

(a) $\dfrac{dy}{dx} = xy^2$

[5 points]

(b) $\dfrac{dy}{dx} + 2y = 5\sin x$

[HINT: It may help to use $\displaystyle\int e^{2x}\sin x\, dx = \frac{1}{5}e^{2x}\left(2\sin x - \cos x\right)$.]

[7 points]

(c) $\dfrac{dy}{dx} + \dfrac{1}{x}y = xy^2$

[8 points]

2006
1. Find the general solutions to the following first-order ODEs:

(a) $\dfrac{dy}{dx} = \dfrac{1+x}{\cos y}$

[6 points]

(b) $\dfrac{dy}{dt} + 3y = \cos t$

[HINT: It may help to use $\displaystyle\int e^{3t}\cos t\, dt = \frac{1}{10}e^{3t}\left(3\cos t + \sin t\right)$.]

[6 points]

(c) $\dfrac{dy}{dx} = \left(\dfrac{y}{x}\right)^2 + 3\left(\dfrac{y}{x}\right) + 1$

[8 points]

2005

1. Find the general solutions to the following first-order ODEs:

(a) $\dfrac{dy}{dx} + 2y = 6e^{-4x}$

[6 points]

(b) $\dfrac{dy}{dx} = ky(1-y)$

[7 points]

(c) $\dfrac{dy}{dx} = \dfrac{y}{x} + \dfrac{x}{y}$

[7 points]

2004

1. Find the general solutions to the following first-order ODEs:

(a) $x\dfrac{dy}{dx} + y = x$

[4 points]

(b) $\dfrac{dy}{dx} = (y+4x)^2$

[HINT: Put $u = y + 4x$. It may help to use $\displaystyle\int \dfrac{du}{u^2 + a^2} = \dfrac{1}{a}\arctan\left(\dfrac{u}{a}\right)$.]

[8 points]

(c) $\dfrac{dy}{dx} + y = -xy^{-1}$

[8 points]

2003

1. Find the general solutions to the following first-order ODEs:

(a) $\sin t \dfrac{dy}{dt} = y\cos t$

[5 points]

(b) $xy\dfrac{dy}{dx} = 2y^2 + 4x^2$

[7 points]

(c) $\dfrac{dy}{dx} = \left(\dfrac{y}{x}\right)^2 + 3\left(\dfrac{y}{x}\right) + 1$

[8 points]

2002
1. Find the general solutions to the following first-order ODEs:

(a) $x^3 \dfrac{dy}{dx} + 3x^2 y = \dfrac{1}{x}$

[5 points]

(b) $x \dfrac{dy}{dx} = (y - x)^3 + y$

[HINT: It may help to make the change of variables $v = y/x$.]

[7 points]

(c) $\dfrac{dy}{dx} + xy = xy^{-1}$

[8 points]